Constraints on the Cosmological Constant due to Scale Invariance

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Abstract: We consider the standard model with local scale invariance. The theory shows exact scale invariance of dimensionally regulated action. We show that massless gauge fields, which may be abelian or non-abelian, lead to vanishing contribution to the cosmological constant in this theory. This result follows in the quantum theory, to all orders in the gauge couplings. However we have not considered contributions higher orders in the gravitational coupling. Similarly we also find that massless fermion fields yield null contribution to the cosmological constant. The effective cosmological constant in this theory is non-zero due to the phenomenon of cosmological symmetry breaking, which also gives masses to all the massive fields, besides generating the Planck mass. We find a simple relationship between the curvature scalar and the vacuum value of the Higgs field in the limit when we ignore all other contributions to the energy density besides the vacuum energy.

1 Introduction

The idea that scale invariance may be implemented as a local symmetry was originally suggested by Weyl [1]. The idea was later revived by several authors [2–10]. A generalized standard model of elementary particles, which displays local scale invariance, has been proposed in Ref. [11]. As expected, the model predicts the existence of the Weyl vector meson. However the Higgs particle disappears from the particle spectrum and acts like the longitudinal model of the Weyl meson. Phenomenological consequences of the Weyl vector

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meson have been studied in Refs. [12–15]. Local scale invariance has also been considered in Refs. [16–25].

However a fundamental problem with the model is the possibility that it may not be consistent quantum mechanically due to scale anomaly. In Ref. [26] it was shown that it is possible to suitably extend the scale symmetry to d dimensions such that the extended symmetry is not anomalous. This was also conjectured earlier in Ref. [27]. This possibility was later studied by several authors [14, 28–33]. In Ref. [31], the authors argued that the scale invariant extension of the Standard model, proposed in Ref. [11], may be consistent quantum mechanically. The authors argued that the model provides a very simple and elegant solution to the standard gauge hierarchy problem. The hierarchy problem is solved since the model contains no physical scalar fields. The scale invariance in this model is broken by the phenomenon of cosmological symmetry breaking [14, 15, 31, 34]. Here the scale invariance is broken by a time dependent classical solution with non-zero space-time curvature. This possibility has also been considered earlier by many authors in the context of global scale invariance [35–47]. The breakdown of scale invariance in this theory generates the gravitational constant, the cosmological constant as well as the masses of the Standard model particles.

The fact that scale anomaly is absent in this theory does not conflict with the standard result that the scale symmetry is anomalous [48–50]. The scale anomaly is found to be absent in this theory only in the case when the symmetry is broken, either spontaneously or cosmologically [14, 15, 31, 34]. In this case the vacuum value of the scalar field or the background curvature provides the scale needed to regulate the action while preserving scale symmetry. However in the limit when these vacuum values vanish, i.e. the symmetry is unbroken, the resulting regulated action becomes ill-defined. This is the limit when the standard result regarding the anomalous nature of the scale symmetry is applicable.

The scale invariant Standard model [11, 14, 15, 31] may be consistent with cosmological observations since it predicts dark energy and dark matter. It may provide a solution to the cosmological constant problem [51–57] due to exact scale invariance, which does not allow a cosmological constant term to appear in the action [14, 30, 47, 58]. Some of the parameters in the model take very small values in order to fit the observed dark energy, the Hubble constant, the gravitational constant and the electroweak symmetry breaking scale. The model provides no explanation for the smallness of these parameters. Despite the presence of such small parameters, the model may not suffer from fine tuning problems if these parameters are stable against quantum corrections [14,30,31]. This might happen due to the exact scale invariance in this theory. In the present paper we consider the constraints that scale invariance imposes on the effective cosmological constant in this theory. Some alternate approaches to solve the cosmological constant problem are discussed in Refs. [51, 59–67].

The action for the scale invariant extension of the Standard Model in d dimensions may be written as [31],

$$\mathcal{S} = \int d^d x\sqrt{-\bar{g}}\bigg[-\frac{\beta}{4}\mathcal{H}^\dagger\mathcal{H}\bar{R}' + \bar{g}^{\mu\nu}(D_\mu\mathcal{H})^\dagger(D_\nu\mathcal{H}) - \frac{1}{4}\bar{g}^{\mu\nu}\bar{g}^{\alpha\beta}(\mathcal{A}^i_{\mu\alpha}\mathcal{A}^i_{\nu\beta}$$

$$
\begin{aligned}
&+ \quad \mathcal{B}_{\mu\alpha}\mathcal{B}_{\nu\beta} + \mathcal{G}^j_{\mu\alpha}\mathcal{G}^j_{\nu\beta})(\bar{R}'^{\,2})^{-\epsilon/4} - \frac{1}{4}\bar{g}^{\mu\rho}\bar{g}^{\nu\sigma}\mathcal{E}_{\mu\nu}\mathcal{E}_{\rho\sigma}(\bar{R}'^{\,2})^{-\epsilon/4} \\
&- \quad \left.\lambda(\mathcal{H}^\dagger\mathcal{H})^2(\bar{R}'^{\,2})^{\,\epsilon/4}\right] + \mathcal{S}_{\text{fermions}}, \qquad (1)
\end{aligned}
$$

where $\mathcal{H}$ is the Higgs doublet, $\mathcal{G}^j_{\mu\nu}$, $\mathcal{A}^i_{\mu\nu}$, $\mathcal{B}_{\mu\nu}$ and $\mathcal{E}_{\mu\nu}$ represent the field strength tensors for the $SU(3)$, $SU(2)$, $U(1)$ and the Weyl vector fields respectively. The superscripts i and j on $\mathcal{A}^i_{\mu\nu}$ and $\mathcal{G}^j_{\mu\nu}$ represent the $SU(2)$ and $SU(3)$ indices respectively. We are implicitly summing over these indices. The symbol $\bar{R}'$ represents the scale covariant curvature scalar,

$$
\bar{R}' = \bar{R} + 4\frac{1-d}{d-2}S^\mu_{;\mu} + (3d-d^2-2)\left(\frac{2}{d-2}\right)^2 f^2 S^\mu S_\mu \qquad (2)
$$

and f is the coupling of the Weyl vector bosons. Here we use the notation of Ref. [68] and denote all quantum gravity variables with a bar. In Eq. 1, β and λ denote the coupling parameters. The regulated fermionic action in d-dimensions is given by,

$$
\begin{aligned}
\mathcal{S}_{\text{fermions}} \quad &= \quad \int d^dx\, e\left(\overline{\Psi}_{\text{L}} i\gamma^\mu\mathcal{D}_\mu\Psi_{\text{L}} + \overline{\Psi}_{\text{R}} i\gamma^\mu\mathcal{D}_\mu\Psi_{\text{R}}\right) \\
&\quad - \int d^dx\, e\,(g_Y\overline{\Psi}_{\text{L}}\mathcal{H}\Psi_{\text{R}}(\bar{R}'^{\,2})^{\,\epsilon/8} + h.c.), \qquad (3)
\end{aligned}
$$

where $e = \det(e_\mu^{\ a})$, $e_\mu^{\ a}e_\nu^{\ b}\eta_{ab} = g_{\mu\nu}$, $\gamma^\mu = e^\mu_{\ a}\gamma^a$ and a, b are Lorentz indices. Here Ψ_{L} is an $SU(2)$ doublet, Ψ_{R} a singlet and g_Y represents a Yukawa coupling. For simplicity we have displayed only one Yukawa coupling term. Furthermore we have displayed the action only for a single family.

The covariant derivative acting on the fermion field is defined by

$$
\mathcal{D}_\mu\Psi_{\text{L,R}} = \left(\tilde{D}_\mu + \frac{1}{2}\omega^{ab}_\mu\sigma_{ab}\right)\Psi_{\text{L,R}}\ , \qquad (4)
$$

where $\tilde{D}_\mu\Psi_{\text{L}} = \partial_\mu - ig\mathbf{T}\cdot\mathbf{A}_\mu - ig'\frac{Y^{\text{L}}_f}{2}B_\mu$, $\tilde{D}_\mu\Psi_{\text{R}} = \partial_\mu - ig'\frac{Y^{\text{R}}_f}{2}B_\mu$ and $\sigma_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$. In Eq. 4, A_μ is the $SU(2)$ field, B_μ the $U(1)$ field, $\mathbf{T}$ represents the SU(2) generators and Y_f's the U(1) hypercharges. Here we have not explicitly displayed the color interactions for quarks, which can be easily added. The spin connection ω^{ab}_μ can be solved in terms of the vierbein,

$$
\omega_{\mu ab} \;=\; \frac{1}{2}(\partial_\mu e_{b\nu} - \partial_\nu e_{b\mu})e_a^{\ \nu} - \frac{1}{2}(\partial_\mu e_{a\nu} - \partial_\nu e_{a\mu})e_b^{\ \nu} - \frac{1}{2}e_a^{\ \rho}e_b^{\ \sigma}(\partial_\rho e_{c\sigma} - \partial_\sigma e_{c\rho})e^c_{\ \mu}\ . \qquad (5)
$$

The spin connection term in covariant derivative makes the fermionic action locally scale invariant. The Weyl vector boson does not couple directly to fermions. However it couples indirectly since the factor $\bar{R}'$ contains contribution from the Weyl meson.

In this paper we shall obtain constraints on the cosmological constant that are imposed due to scale invariance. The model does not permit a cosmological constant term in the action. We expect that classically the trace of the energy momentum tensor would vanish

due to conservation of the scale current J^μ, i.e.

$$(J^\mu)_{;\mu} = T^\mu_\mu = 0, \tag{6}$$

where $T_{\mu\nu}$ is the energy momentum tensor. This would normally lead to direct constraints on the energy density of the matter fields. In particular this would imply that the vacuum energy density is zero. However due to non-minimal coupling of the matter fields to gravity this does not necessarily follow. The scale invariance is broken by the phenomenon which we refer as the cosmological symmetry breaking [14, 34]. This leads to a non-zero value for the cosmological constant. Since the model does not permit a cosmological constant term, the value of this parameter generated due to symmetry breaking is a prediction of the model.

In the present paper we shall determine the constraints that scale invariance imposes on the cosmological constant in the full quantum theory. We shall work within the framework of canonical quantization. The fact that we can regulate the action while preserving scale invariance implies that scale symmetry is exact in the full quantum theory. A basic observable of cosmological interest is the curvature scalar R. The curvature scalar is related to the energy momentum tensor by the Einstein's equations. As we shall see, in the present theory, it is also related to the matter fields by the Higgs field equation of motion. We point out that this equation is valid in the full quantum theory as the Heisenberg operator equation [69]. All operators arising in this equation are well defined since the equation is obtained from the regulated action. It turns out that this equation provides us with a much simpler equation for determining R.

In our analysis we shall focus primarily on the contributions due to the Standard Model fields. We shall not explicitly include the contributions due to the Weyl vector meson. These depend on another unknown parameter f. The contribution of Weyl vector meson to the cosmological constant will be small as long as this parameter is sufficiently small. However we shall not consider the precise observational constraint on this parameter in this paper. Furthermore we shall also ignore quantum gravity effects.

In the next section we consider a simple system of real scalar field coupled to gravity. In section 3 we analyse the energy momentum tensor in the Standard Model.

2 A Toy Model

In order to get oriented we first consider a simple system of a real scalar field coupled to gravity. The action may be written as

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{\lambda}{4}\Phi^4 - \frac{\beta}{8}\Phi^2 R\right], \tag{7}$$

The Einstein's equations are

$$\Phi^2\left[-\frac{1}{2}g_{\alpha\beta}R + R_{\alpha\beta}\right] + (\Phi^2)_{;\lambda;\kappa}\left[-\frac{1}{2}(g^\lambda_\alpha g^\kappa_\beta + g^\kappa_\alpha g^\lambda_\beta) + g_{\alpha\beta}g^{\lambda\kappa}\right] = \frac{4}{\beta}T_{\alpha\beta}, \tag{8}$$

where

$$T_{\alpha\beta} = \partial_\alpha \Phi \partial_\beta \Phi - g_{\alpha\beta}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{\lambda}{4}\Phi^4\right]. \tag{9}$$

The equation of motion for the scalar field is given by,

$$g^{\mu\nu}\Phi_{;\mu;\nu} + \frac{\beta}{4}\Phi R + \lambda\Phi^3 = 0. \tag{10}$$

We point out that $T_{\alpha\beta}$ is not the total energy momentum tensor since it does not include the contribution from the term proportional to β. We shall refer to $T_{\alpha\beta}$ as the truncated energy momentum tensor. In this theory the scalar field is not separable from the gravitational action. Hence the total energy momentum tensor would necessarily involve contributions from the gravitational sector also. We may determine the energy momentum tensor by varying the action with respect to the gravitational field and identifying the terms linear in the fluctuations $\delta g_{\mu\nu}$. The corresponding tensor, and hence its trace, vanishes trivially by the Einstein's equations.

The action given in Eq. 7 displays the phenomenon of cosmological symmetry breaking. We assume a FRW metric with the curvature parameter k set to zero, for simplicity. We find a classical solution with the scalar field Φ_0 equal to a constant and

$$\frac{\beta}{4}R = -\lambda\Phi_0^2. \tag{11}$$

The FRW scale parameter is found to be,

$$a(t) = a_0 \exp(H_0 t) \tag{12}$$

with

$$\Phi_0 = \sqrt{\frac{3\beta}{\lambda}}H_0, \tag{13}$$

where H_0 is identified with the Hubble constant. This is a time dependent solution to the equations of motion. If we assume the Big Bang model for the universe then all physical phenomena take place in this background. Here we have generated an effective cosmological constant equal to $\lambda\Phi_0^4/4$ due to the non-minimal coupling of the scalar field with gravity. Specifically we mean the term proportional to $\Phi^2 R$ in the action. In order to study any process we need to make a quantum expansion around the classical background Φ_0. We point out that since we are not making an expansion around the minimum of the potential, the lowest energy state is not the true ground state of the theory. In order to obtain the curvature scalar in the full quantum theory, we need to take the expectation value of Eq. 8 with respect to the lowest energy state. We shall consider the adiabatic limit where the time dependence of the classical solution is very slow. Hence we shall drop terms which are higher order in the derivative of the metric. These will introduce additional powers of the Hubble constant H_0 which is the smallest energy scale in the theory. In fact all other energy

scales are related to this fundamental scale but these will involve factors of $1/\lambda$ or β which we assume to be large. We shall also ignore higher order corrections in quantum gravity.

We next consider the Einstein's equation for the toy model. It is useful to define the tensor $\tilde{T}_{\mu\nu}$, which collects all the terms in the Einstein's equation, besides the term proportional to the Einstein tensor $(R_{\mu\nu} - g_{\mu\nu}R/2)$. We have

$$\tilde{T}_{\alpha\beta} = T_{\alpha\beta} - \frac{\beta}{4}(\Phi^2)_{;\lambda;\kappa}\left[-\frac{1}{2}(g^\lambda_\alpha g^\kappa_\beta + g^\kappa_\alpha g^\lambda_\beta) + g_{\alpha\beta}g^{\lambda\kappa}\right]. \tag{14}$$

We are interested in evaluating trace of this tensor.

We expand the full quantum metric, denoted by $\bar{g}_{\mu\nu}$, around the classical solution, $g_{\mu\nu}$,

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}. \tag{15}$$

Here we treat gravity classically and only the matter fields are quantized. Hence in the expansion of the action we shall only keep terms linear in the field $h_{\mu\nu}$. We may express the terms linear in $h_{\mu\nu}$ as,

$$\delta S = \frac{1}{2}\int d^4x\sqrt{-g}\left[\frac{\beta}{4}\Phi^2\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) - \tilde{T}_{\mu\nu}\right]h^{\mu\nu} \tag{16}$$

The Einstein's equations are obtained by demanding that the terms inside the square brackets vanish. We expand the field Φ around the classical solution Φ_0,

$$\Phi = \Phi_0 + \phi\,. \tag{17}$$

We point out that the classical solution Φ_0 is constant. Since we are interested in a quantum calculation we must suitably regulate the action. We use dimensional regularization, which introduces an extra factor $(R^2)^{\epsilon/4}$ in self coupling term proportional to λ, in analogy with the regulated action shown in Eq. 1.

Before we proceed we note that the term proportional to $(\Phi^2)_{;\lambda;\kappa}$ in Eq. 14 does not contribute to the cosmological constant. The reason is the following - the contribution of this term to the action is proportional to

$$\int d^dx\sqrt{-g}\left[(\Phi^2)_{;\lambda}\right]^{;\lambda}h = \int d^dx\left[\sqrt{-g}(\Phi^2)_{;\lambda}\right]^{,\lambda}h\,,$$

where $h = h^\mu_\mu$. After integrating by parts this term becomes proportional to a derivative of h which means that it can contribute only if the external graviton has a non-zero momentum. Hence it contributes zero to the cosmological constant. Same argument applies to all terms in the Lagrangian density of the form $\sqrt{-g}(V_\alpha)_{;\beta}h^{\alpha\beta}$, where V_α is a vector. A term in the Lagrangian density of the form $\sqrt{-g}(V_\alpha)_{;\beta}g^{\alpha\beta}$, where $g^{\alpha\beta}$ is the classical metric, vanishes trivially by integration by parts. Hence we find the general result that all terms in the energy momentum tensor of the form $(V_\alpha)_{;\beta}$ give zero contribution to the cosmological constant.

We next consider the trace of $T_{\alpha\beta}$. We may express the trace as

$$T^\alpha_\alpha = \left(1 - \frac{\epsilon}{2}\right)\left[-(\Phi\Phi^{;\mu})_{;\mu} + \Phi\Phi^{;\mu}_{;\mu} + \lambda\Phi^4(R^2)^{\epsilon/4}\right] + \dots . \tag{18}$$

Here we have not explicitly displayed some terms which are proportional to an overall derivative and hence do not contribute to cosmological constant. The first term on the right side would also not contribute to the cosmological constant for the same reason. Taking the trace of the Einsteins's equation, Eq. 8, in d dimensions, we find,

$$g^{\mu\nu}\Phi\Phi_{;\mu;\nu} + \frac{\beta}{4}\Phi^2 R + \lambda\Phi^4(R^2)^{\epsilon/4} + ... = 0. \tag{19}$$

where we have used Eq. 18. Here, once again, we have dropped the surface terms which do not contribute to the cosmological constant. The result of Eq. 19, actually follows much more directly from the equation of motion of the scalar field, i.e., Eq. 10 suitably generalized to d-dimensions. This is a consequence of the non-minimal coupling of the scalar field to gravity.

Classically the curvature scalar R is determined by Eq. 11. In the quantum theory we need to compute the higher order corrections to this result. We may choose the renormalization scheme such that the expectation value of Φ in the lowest energy state,

$$< \Phi >= \Phi_0 \tag{20}$$

and $< \phi >= 0$ to all orders. This implies that

$$< \Phi >^2 = -\frac{\beta}{4\lambda} R\,, \tag{21}$$

to all orders. This equation directly fixes the value of the curvature R in terms of the renormalized parameters β, λ and $< \Phi >$. It is valid as long as we include contributions only due to vacuum energy.

We may impose local scale invariance on this toy model by introducing the Weyl vector meson S_μ. In this case the real scalar field disappears from the particle spectrum and acts like the longitudinal mode of the Weyl meson. We again expand around a classical solution $\Phi = \Phi_0 + \phi$, with Φ_0 equal to a constant. Furthermore lets assume that $< S_\mu >= 0$, i.e. the expectation value of the Weyl meson field in the lowest energy state is exactly equal to zero. As we expand around the classical solution, the one point function $< \phi >$ has to vanish in this case since the scalar particle is unphysical. Hence we directly obtain Eq. 21 at all orders, which fixes R in terms of other renormalized parameters. We study the implications of this in the next section where we study the model defined by Eq. 1.

3 Constraints on the Cosmological Constant

We now obtain the constraints on the cosmological constant due to scale invariance for the standard model defined by Eq. 1. Here we shall treat gravity classically. The Einstein's equation may be written as,

$$\mathcal{H}^\dagger\mathcal{H}\left[-\frac{1}{2}g_{\alpha\beta}R+R_{\alpha\beta}\right]+(\mathcal{H}^\dagger\mathcal{H})_{;\lambda;\kappa}\left[-\frac{1}{2}(g^\lambda_\alpha g^\kappa_\beta+g^\kappa_\alpha g^\lambda_\beta)+g_{\alpha\beta}g^{\lambda\kappa}\right]=\frac{2}{\beta}T_{\alpha\beta}, \tag{22}$$

where the truncated energy momentum tensor, $T_{\alpha\beta}$, contains contributions due to all the terms excluding the term proportional to β in the action. We are interested in extracting the effective cosmological constant or equivalently the curvature scalar R from this equation. The contribution of the second term on the left hand side of Eq. 22 to R vanishes in analogy to the corresponding term in the toy model considered in Section 2.

3.1 Pure Higgs Field

Let us first consider the scalar Higgs field coupled to gravity. We ignore all other fields for now. This is very similar to the toy model considered in last section, except that we also take into account the regulator and we are dealing with a complex doublet. If we include contribution only from the Higgs field we find,

$$\begin{aligned}T_{\alpha\beta} &= (D_\alpha\mathcal{H})^\dagger D_\beta\mathcal{H}+(D_\beta\mathcal{H})^\dagger D_\alpha\mathcal{H}-g_{\alpha\beta}\left[g^{\mu\nu}(D_\mu\mathcal{H})^\dagger D_\nu\mathcal{H}-\lambda(\mathcal{H}^\dagger\mathcal{H})^2(R^2)^{\epsilon/4}\right]\\ &\quad -\epsilon\lambda(\mathcal{H}^\dagger\mathcal{H})^2(R^2)^{(\epsilon-2)/4}R_{\alpha\beta}+\dots\,.\end{aligned} \tag{23}$$

Here the '...' denotes terms which are proportional to an overall derivative besides terms proportional to the Weyl meson coupling f. As mentioned in the introduction we shall not explicitly consider the contribution due to Weyl meson in this paper. The terms in $T_{\alpha\beta}$ which are of the form $(V_\alpha)_{;\beta}$, where V_α is a vector, do not contribute to the cosmological constant. These terms are proportional to an overall derivative and as shown in Section 2, their contributions vanish.

The trace of the remaining terms, explicitly displayed in Eq. 23, may be expressed as

$$T^\alpha_\alpha = \left(1-\frac{\epsilon}{2}\right)\left[-2D_\mu(g^{\mu\nu}\mathcal{H}^\dagger D_\nu\mathcal{H})+2g^{\mu\nu}\mathcal{H}^\dagger D_\mu D_\nu\mathcal{H}+4\lambda(\mathcal{H}^\dagger\mathcal{H})^2(R^2)^{\epsilon/4}\right]+\dots\,. \tag{24}$$

The first term on the right hand side is a total derivative and can be ignored. If we take the trace of the Einstein's equation, Eq. 22, and substitute T^α_α into the resulting equation we obtain,

$$\mathcal{H}^\dagger D^\mu D_\mu\mathcal{H}+\frac{\beta}{4}R\mathcal{H}^\dagger\mathcal{H}+2\lambda(\mathcal{H}^\dagger\mathcal{H})^2(R^2)^{\epsilon/4}+...=0\,. \tag{25}$$

As before, this result can be obtained much more directly by using the equation of motion

of the Higgs field coupled to gravity, i.e.,

$$D^\mu D_\mu \mathcal{H} + \frac{\beta}{4} R\mathcal{H} + 2\lambda \mathcal{H}(\mathcal{H}^\dagger \mathcal{H})(R^2)^{\epsilon/4} = 0\,. \tag{26}$$

3.2 Pure Gauge Fields

Next, we consider the contribution due to a pure non-abelian gauge field (the calculation for an abelian gauge field is similar except the fact that there is no Faddeev-Popov ghost term). The canonical quantization of non-abelian gauge fields is discussed in Refs. [70–72]. The corresponding action may be written as,

$$\mathcal{S}_{\rm A} = \int d^d x \sqrt{-g} \left[-\frac{1}{4} g^{\mu\nu} g^{\alpha\beta} (\mathcal{A}^i_{\mu\alpha} \mathcal{A}^i_{\nu\beta})(R^2)^{-\epsilon/4} + \mathcal{L}^{gf} + \mathcal{L}^{\rm FPG} \right], \tag{27}$$

where $\mathcal{A}^i_{\mu\nu}$ represents the field strength tensor of the gauge field and the superscript i represents the internal index. Summation over i is implicit. The gauge fixing and the Faddeev-Popov ghost Lagrangian are given by,

$$\mathcal{L}^{gf} = -\frac{1}{2\xi} g^{\mu\nu} g^{\alpha\beta} D_\mu A^a_\nu D_\alpha A^a_\beta (R^2)^{-\epsilon/4} \tag{28}$$

and

$$\mathcal{L}^{\rm FPG} = c^\dagger_a g^{\alpha\beta} D_\alpha \left[\delta_{ab} D_\beta - g C_{abc} A^c_\beta \right] c_b \tag{29}$$

respectively. Here ξ is the gauge parameter, c_a, $c^\dagger_a$ the ghost fields, D_μ the covariant derivative, g the gauge coupling and C_{abc} represents the gauge group structure constants.

The contribution of the kinetic energy term to the energy momentum tensor is given by

$$\begin{aligned} T^{\rm A}_{\sigma\rho} &= \frac{1}{4} g_{\sigma\rho} [g^{\mu\nu} g^{\alpha\beta} \mathcal{A}^i_{\mu\alpha} \mathcal{A}^i_{\nu\beta} (R^2)^{-\epsilon/4}] - [g^{\alpha\beta} \mathcal{A}^i_{\sigma\alpha} \mathcal{A}^i_{\rho\beta} (R^2)^{-\epsilon/4}] \\ &+ \frac{\epsilon}{4} [g^{\mu\nu} g^{\alpha\beta} \mathcal{A}^i_{\mu\alpha} \mathcal{A}^i_{\nu\beta} (R^2)^{-(\epsilon+2)/4}] R_{\sigma\rho} \\ &+ \frac{\epsilon}{4} [g^{\mu\nu} g^{\alpha\beta} \mathcal{A}^i_{\mu\alpha} \mathcal{A}^i_{\nu\beta} (R^2)^{-(\epsilon+2)/4}]_{;\gamma;\delta} \left[-\frac{1}{2} (g^\gamma_\sigma g^\delta_\rho + g^\delta_\sigma g^\gamma_\rho) + g_{\sigma\rho} g^{\gamma\delta} \right]. \end{aligned} \tag{30}$$

Once again we have ignored terms proportional to the Weyl meson coupling f. In order to determine the contribution to the cosmological constant we consider the trace of the tensor $T^{\rm A}_{\sigma\rho}$. We find that when we take the trace of the first three terms on the right hand side cancel identically in d dimensions. This leaves us with the last, the fourth term. This term of course vanishes classically when we take the limit $\epsilon \to 0$. However it may contribute at loop orders.

The trace of the fourth term on the right hand side of Eq. 30 may be written as,

$$\frac{\epsilon}{4} [g^{\mu\nu} g^{\alpha\beta} \mathcal{A}^i_{\mu\alpha} \mathcal{A}^i_{\nu\beta} (R^2)^{-(\epsilon+2)/4}]^{;\gamma}_{;\gamma} (d-1). \tag{31}$$

This term is of the form $(V^\mu)_{;\mu}$ and hence does not contribute to the cosmological constant.

Hence we find that the kinetic energy terms for the vector fields do not contribute to the cosmological constant.

The contribution due to the gauge fixing term may be written as,

$$\begin{aligned} T^{gf}_{\alpha\beta} &= \frac{1}{2\xi}\left[g_{\alpha\beta}(D\cdot A^a)^2 - 4(D\cdot A^a)(D_\alpha A^a_\beta) + \epsilon(D\cdot A^a)^2\frac{R_{\alpha\beta}}{R}\right](R^2)^{-\epsilon/4} \\ &- \frac{\epsilon}{2\xi}g_{\alpha\beta}D^\mu D_\mu\left[(D\cdot A^a)^2\frac{(R^2)^{-\epsilon/4}}{R}\right] + \frac{\epsilon}{2\xi}D_\alpha D_\beta\left[(D\cdot A^a)^2\frac{(R^2)^{-\epsilon/4}}{R}\right] \\ &+ \frac{2}{\xi}D_\alpha[A^a_\beta(D\cdot A^a)(R^2)^{-\epsilon/4}] - \frac{1}{\xi}g_{\alpha\beta}D^\nu[A^a_\nu(D\cdot A^a)(R^2)^{-\epsilon/4}], \end{aligned} \tag{32}$$

where $D\cdot A^a \equiv D^\mu A^a_\mu$. When we take the trace of this expression we find that all the terms either cancel out or are equal to total derivatives. After integration by parts such total derivative terms do not contribute to the cosmological constant. Hence the contribution of the gauge fixing term to cosmological constant also vanishes.

Finally we consider the contribution due to the Faddeev-Popov ghost term. Its contribution to the energy momentum tensor may be expressed as

$$T^{\rm FPG}_{\mu\nu} = -g_{\mu\nu}c^\dagger_a D^\alpha\left[\delta_{ab}D_\alpha - gC_{abc}A^c_\alpha\right]c_b + 2c^\dagger_a D_\mu\left[\delta_{ab}D_\nu - gC_{abc}A^c_\nu\right]c_b. \tag{33}$$

The trace of this term vanishes after using the ghost field equations of motion.

Hence we find that pure gauge fields yield null contribution to the cosmological constant. We note the crucial use of the scale invariant dimensional regulator used in arriving at this result. This implies that all massless gauge fields, abelian or non-abelian, do not contribute to the trace of the energy momentum tensor. The result applies directly to QED and QCD gauge fields and is exact at all orders in the gauge coupling. However we have not considered terms higher order in the gravitational coupling and the Weyl meson coupling f.

3.3 Massless Dirac Fermions

The action for massless Dirac fermions in d dimensions may be written as

$$\mathcal{S}_\Psi = \int d^dx e\,(\overline{\Psi}i\gamma^\mu\mathcal{D}_\mu\Psi)\,. \tag{34}$$

Here we ignore all the gauge fields and the covariant derivative includes only the contribution due to gravity. The energy momentum tensor is found to be,

$$\begin{aligned} T^\Psi_{\alpha\beta} &= -g_{\alpha\beta}(\overline{\Psi}i\gamma^\rho\mathcal{D}_\rho\Psi) + \frac{1}{2}\overline{\Psi}i\gamma_\alpha\mathcal{D}_\beta\Psi + \frac{1}{2}\overline{\Psi}i\gamma_\beta\mathcal{D}_\alpha\Psi \\ &+\frac{g_{\alpha\beta}}{2}(\overline{\Psi}i\gamma_\rho\Psi)^{;\rho} - \frac{1}{4}(\overline{\psi}i\gamma_\beta\psi)_{;\alpha} - \frac{1}{4}(\overline{\psi}i\gamma_\alpha\psi)_{;\beta}. \end{aligned} \tag{35}$$

The terms in the second line, which represent total covariant derivatives, do not contribute to the cosmological constant. The terms in the first line are found to be zero by using the

equation of motion

$$i\gamma^\mu \mathcal{D}_\mu \Psi = 0. \tag{36}$$

The above result also applies directly if we include gauge interactions of fermions. The only modification here is that the covariant derivative also includes the contribution from the corresponding gauge fields. Hence we find that we get zero contribution to the cosmological constant from both the strong and electromagnetic interactions as long as we do not include the mass terms for the fermions or equivalently the Yukawa interactions terms. We include these terms below along with the electroweak gauge fields.

3.4 The Scale Invariant Standard Model

We now consider the full action given in Eq. 1. The scalar field equation of motion is given by,

$$D_\mu D^\mu \mathcal{H} + 2\lambda(\mathcal{H}^\dagger \mathcal{H})\mathcal{H}(R^2)^{\epsilon/4} + g_Y \bar{\Psi}_{\mathrm{R}}\Psi_{\mathrm{L}}(R^2)^{\epsilon/8} + \frac{\beta}{4}\mathcal{H}R = 0\,. \tag{37}$$

Here we have not explicitly displayed the contribution due to the gauge fixing terms and the Faddeev-Popov ghost terms. The equations of motion for the $SU(2)$ doublet and singlet fermions may be written as,

$$\begin{aligned} i\gamma^\mu D_\mu \Psi_{\mathrm{R}} - g_Y \mathcal{H}^\dagger \Psi_{\mathrm{L}}(R^2)^{\epsilon/8} &= 0, & (38)\\ i\gamma^\mu D_\mu \Psi_{\mathrm{L}} - g_Y \mathcal{H} \Psi_{\mathrm{R}}(R^2)^{\epsilon/8} &= 0. & (39)\end{aligned}$$

The truncated energy momentum tensor $T_{\alpha\beta}$ is found to be

$$\begin{aligned} T_{\alpha\beta} = & -g_{\alpha\beta}\Big[(D_\mu\mathcal{H})^\dagger D^\mu\mathcal{H} + \bar{\Psi}_{\mathrm{L}} i\gamma^\rho D_\rho \Psi_{\mathrm{L}} + \bar{\Psi}_{\mathrm{R}} i\gamma^\rho D_\rho \Psi_{\mathrm{R}} - \lambda(\mathcal{H}^\dagger\mathcal{H})^2(R^2)^{\epsilon/4} \\ & -g_Y(\bar{\Psi}_{\mathrm{L}}\mathcal{H}\Psi_{\mathrm{R}} + h.c.)(R^2)^{\epsilon/8}\Big] + (D_\alpha\mathcal{H})^\dagger D_\beta\mathcal{H} + (D_\beta\mathcal{H})^\dagger D_\alpha\mathcal{H} \\ & +\frac{1}{2}\bar{\Psi}_{\mathrm{L}} i\gamma_\alpha D_\beta\Psi_{\mathrm{L}} + \frac{1}{2}\bar{\Psi}_{\mathrm{R}} i\gamma_\alpha D_\beta\Psi_{\mathrm{R}} + \frac{1}{2}\bar{\Psi}_{\mathrm{L}} i\gamma_\beta D_\alpha\Psi_{\mathrm{L}} + \frac{1}{2}\bar{\Psi}_{\mathrm{R}} i\gamma_\beta D_\alpha\Psi_{\mathrm{R}} \\ & -\lambda\epsilon(\mathcal{H}^\dagger\mathcal{H})^2(R^2)^{(\epsilon-2)/4}R_{\alpha\beta} - \frac{\epsilon g_Y}{2}(\bar{\Psi}_{\mathrm{L}}\mathcal{H}\Psi_{\mathrm{R}} + h.c.)(R^2)^{(\epsilon-4)/8}R_{\alpha\beta} + \ldots, \end{aligned} \tag{40}$$

where we have not explicitly displayed terms whose contribution to the cosmological constant vanishes. These include the kinetic energy terms for the gauge fields. We have also not displayed the contributions due to the gauge fixing terms and the Faddeev-Popov ghost terms. The trace of the truncated energy momentum tensor is found to be,

$$T^\alpha_\alpha = -(1-\epsilon/2)\left[2(D_\mu\mathcal{H})^\dagger D^\mu\mathcal{H} - g_Y(\bar{\Psi}_{\mathrm{L}}\mathcal{H}\Psi_{\mathrm{R}} + h.c.)(R^2)^{\epsilon/8} - 4\lambda(\mathcal{H}^\dagger\mathcal{H})^2(R^2)^{\epsilon/4}\right] + \ldots, \tag{41}$$

where we have used the fermion equations of motion to eliminate the fermion kinetic energy terms. We now substitute T^α_α into the trace of the Einstein's equation, Eq. 22. We obtain

a relatively simple expression for the trace of the truncated energy momentum tensor or equivalently the curvature scalar. Many of the terms cancel exactly, as expected from scale invariance. In fact the non-zero contribution arises primarily due to the non-minimal coupling of the Higgs field to gravity.

In this model the cosmological constant is generated by the phenomenon of cosmological symmetry breaking. Again we assume a FRW metric with curvature parameter $k = 0$ and we expand the scalar field around its classical solution, $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}'$, with,

$$\mathcal{H}_0^\dagger \mathcal{H}_0 = -\frac{\beta R}{4\lambda}. \tag{42}$$

We assume that $< S_\mu >= 0$, i.e. the expectation value of the Weyl meson field in the lowest energy state is identically equal to zero. Following the argument given at the end of Section 2, the one point function, $< \mathcal{H}' >$, must vanish in this theory. This follows by gauge invariance. Since the Higgs field is the longitudinal mode of the Weyl meson, its one point function must vanish as long as $< S_\mu >= 0$. This implies that

$$< \mathcal{H} >= \mathcal{H}_0 = \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{43}$$

exactly to all order in the perturbation theory. Here

$$v = \sqrt{-\frac{\beta R}{4\lambda}}. \tag{44}$$

The parameter $\mathcal{H}_0$ in Eq. 43 is fixed by its relationship to the W boson mass,

$$M_{\mathrm{W}}^2 = g^2 (\mathcal{H}_0^\dagger \mathcal{H}_0), \tag{45}$$

where g is the gauge coupling. In Eq. 43 we have used the electroweak symmetry to set the first entry of the classical scalar doublet to be zero. This is analogous to the choice normally made in analysing the electroweak symmetry breaking in the standard Weinberg-Salam model. We, therefore, find that the curvature scalar and hence the effective cosmological constant is determined by Eqs. 43 and 44 in the standard model with local scale invariance. In Eq. 44, β and λ are the renormalized parameters. The parameter $1/\beta$ is effectively the gravitational coupling. Classically it is related to the Planck mass by the formula,

$$\beta \mathcal{H}_0^\dagger \mathcal{H}_0 = \frac{M_{\mathrm{PL}}^2}{4\pi}. \tag{46}$$

Hence this parameter is fixed by the value of the known gravitatonal coupling.

The coupling parameter λ does not directly relate to any scattering cross section or the mass of any particle since the Higgs meson is not a physical particle in this theory. It contributes directly to the value of the curvature scalar through Eqs. 43 and 44. We may fix this parameter by the observed value of the cosmological constant or equivalently the value

of the curvature scalar R. Hence we find that there is sufficient freedom in the model in order to fit the observed value of the effective cosmological constant, despite the fact that we are not allowed to add a cosmological constant term to the action.

The model solves the gauge hierarchy problem since it does not contain any fundamental scalar field in the physical spectrum [31]. However the model introduces three parameters, $1/\beta$, λ and f, which take very small values. The model offers no insight into why these are driven to such small values. Furthermore it is possible that the model may suffer from fine tuning problems at higher orders in perturbation theory due to the small values of these parameters. The parameter $1/\beta$ is equivalent to the gravitational coupling and its smallness is well known. The detailed renormalization of this parameter is beyond the scope of the present paper. Even at one loop we do not expect the gravitational sector of the theory to be renormalizable. Hence we either need to treat gravity classically or perform quantum gravity calculations treating it as an effective theory, by introducing an infinite number of counter terms [73]. Nevertheless we do not expect any fine tuning in the gravitational coupling. As we shall see below the remaining two parameters are also intrinsically tied to the gravitational sector of the theory.

The renormalization of the self coupling λ is constrained by the requirement that the one point function of the Higgs field must vanish as long as the expectation value of the Weyl meson field, $< S_\mu >$, is zero. This requirement, imposed by gauge invariance, relates the counterterm corresponding to the self coupling to the gravitational counter term action. Hence the renormalization of λ is intrisically tied to the renormalization of the gravitational field. In the absence of a consistent theory of quantum gravity we may simply ignore higher order corrections in $1/\beta$. In this case the theory is renormalizable and we can consistently study the renormalization group evolution of λ. We point out that we do not expect any fine tuning in this parameter at loop orders since it is expected to depend logarithmically on the renormalization scale. A small value of λ may be natural due to the triviality of the pure scalar field theory. If we ignore all other fields then this parameter must vanish. However it is possible that quantum gravity effects might lead to a small value of this parameter since they impose an effective ultraviolet cutoff on the theory.

Finally we consider the Weyl meson coupling f, which we have also assumed to be very small. This parameter can be fixed by considering the scattering of Weyl mesons and gravitons. We cannot consider its scattering with Standard model particles, since all these couplings vanish. Hence we find that the renormalization of the coupling f is also tied primarily with the gravitational field. We expect that loop corrections to f are likely to be highly suppressed since these will involve additional powers of f and/or the gravitational coupling.

4 Conclusions

In this paper we have considered an extension of the standard model of particle physics with local scale invariance. We have shown that scale invariance imposes considerable constraints on the effective cosmological constant or the trace of the energy momentum tensor. The

contribution of massless gauge fields and fermions to the cosmological constant is shown to be identically zero. This implies that QCD (or QED) in the limit of zero fermion masses would yield a vanishing contribution to the cosmological constant. The result follows exactly to all orders in gauge coupling. However we have not considered higher orders in gravitational coupling or the Weyl coupling f. In hindsight the vanishing of the cosmological constant for massless gauge fields and fermions is not very surprising. It follows directly from the scalar field equation of motion, Eq. 37, which provides us with another equation, besides the Einstein's equations, to compute the curvature scalar R. The massless gauge fields or fermions do not contribute to this equation and hence do not contribute to R and equivalently to the cosmological constant.

The model produces non-zero effective cosmological constant due to the phenomenon of cosmological symmetry breaking. We have provided a formalism to compute the curvature scalar R in this model. The final result is very simple and given by the Eqs. 43 and 44. These relate R to other parameters in the theory.

The model contains some parameters $1/\beta$ and λ which take very small values. We have also assumed that the Weyl meson coupling f is very small. The parameter $1/\beta$ is related to the gravitational constant and its smallness is well known. We have shown that quantum corrections to λ are constrained by local scale invariance. In any case this scalar field self coupling parameter is expected to depend logarithmically on the renormalization scale and hence is not likely to suffer from any fine tuning problems. The loop corrections to f are also suppressed by additional powers of the coupling f besides the gravitational coupling. Its renormalization is intrinsically tied to quantum gravity effects since S_μ does not couple directly to Standard model particles. Hence we are unable to properly address the issue of renormalization group evolution of this coupling in the absence of a consistent theory of quantum gravity. The effective cosmological constant or equivalently the curvature scalar is related to other parameters of the theory by the formulas, Eqs. 43 and 44. Hence the curvature scalar is not expected to suffer from any fine tuning problems as long as the remaining parameters do not require any fine tuning. If we ignore quantum gravity effects we expect that this is true in the present model.

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